Evaluation of Electromagnetic Scattering by Conducting Bodies of Revolution with Discontinuous Currents

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In this work the electromagnetic radiation from conducting bodies of revolution with discontinuities in the equivalent surface currents is investigated. The simulations are based on the Combined Field Integral Equation evaluated by the Method of Moments. A strategy for implementing an integral equation formulation without restrictions over the testing functions is presented. The procedure is applied in the analysis of the radiation from a circular horn antenna and numerical results are compared with experimental data.

***Index Terms*—Electric and magnetic field integral equations, electromagnetic scattering by bodies of revolution, Galerkin technique, method of moments.**

I. Introduction

The electromagnetic scattering by conducting objects has been a subject of intense investigation for many years. Research efforts have led to the development of a large number of analysis tools and modeling techniques. Among these methods, surface integral equations are probably the most suitable ones for numerical simulations. When dealing with open conducting shells, generally the most efficient method is based on the solution of an Electric Field Integral Equation (EFIE). For closed conducting bodies the Combined Field Integral Equation (CFIE) is largely employed due its capacity of reducing spurious resonances [1]-[2]. Just over the metallic surface (S) of the object, the CFIE is represented by the linear combination αTE $+\beta$ NH, where

$$\mathrm{TE} \rightarrow 2\int_S \mathrm{W}(\boldsymbol{r})\cdot\left(\eta\mathbf{L}(\mathbf{J})+\mathbf{K}(\mathbf{M})+\mathbf{E}^{\mathrm{i}}\right)ds = -\int_S \mathrm{W}(\boldsymbol{r})\cdot(\hat{\boldsymbol{n}}\times\mathbf{M})\,ds \,, \quad (1)$$

$$\mathrm{NH} \rightarrow 2\int_S \mathrm{W}(\boldsymbol{r})\cdot\hat{\boldsymbol{n}}\times\left(\mathbf{L}(\mathbf{M})/\eta-\mathbf{K}(\mathbf{J})+\mathbf{H}_0^{\mathrm{i}}\right)ds = \int_S \mathrm{W}(\boldsymbol{r})\cdot\mathbf{J}\,ds \,, \quad (2)$$

$$\mathbf{L}(\mathbf{X}) = j/(4\pi k)\iint_{S'}\left[k^2\,\mathbf{X}(\mathbf{r}')\,\mathrm{G}(\mathbf{r},\mathbf{r}')-\nabla'\,\mathbf{X}(\mathbf{r}')\,\nabla'\,\mathrm{G}(\mathbf{r},\mathbf{r}')\right]ds' \,, \quad (3)$$

$$\mathbf{K}(\mathbf{X}) = -\hat{n}\times\mathbf{X}(\mathbf{r})/2 + 1/(4\pi)\iint_{S'}\left[\mathbf{X}(\mathbf{r}')\times\nabla'\,\mathrm{G}(\mathbf{r},\mathbf{r}')\right]ds', \quad (4)$$

α and β are scalar weights, $\hat{n}$ is the unit outward normal vector to S, W is the test function, **X** is either the equivalent electric (**J**) or magnetic (**M**) surface current, $\mathbf{E}^{\mathrm{i}}$ and $\mathbf{H}^{\mathrm{i}}$ are the electric and magnetic incident fields radiated by external sources, k and η are the wavelength number and intrinsic impedance of the medium, respectively, and G(**r**, **r′**) is the free space Green's function.

In the numerical solution of the CFIE by the Method of Moments (MoM) with a Galerkin approach, the basis functions used to represent **J** and **M** are identical to the weight (test) functions. However, for Bodies of Revolution (BOR), as that illustrated in Fig. 1, the normal term of the CFIE [NH in (2), where a cross product by $\hat{n}$ is present] contains a reciprocal interchange between the test functions in $\hat{t}$ and $\hat{\phi}$ directions. Furthermore, if the $\hat{t}$ and $\hat{\phi}$ components of **J** or **M** have different local behaviors (e.g., discontinuities at junctions) the numbers of basis functions used to represent these $\hat{t}$ and $\hat{\phi}$ components will probably be different from each other. Consequently, in the Galerkin approach each TE and NH terms of the CFIE will be tested by a different number of test functions to provide the MoM linear system. This leads to difficulties in the numerical implementation of the CFIE, which are enhanced by the fact that **K** presents more severe singularities than **L** [3].

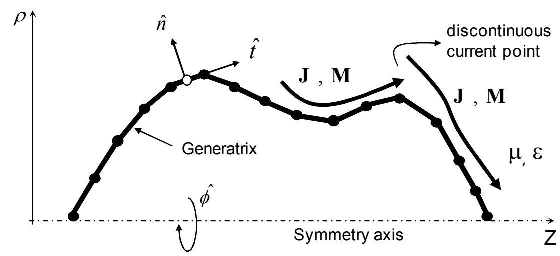

Fig. 1. Generating curve of a body of revolution.

Although CFIE has been extensively used for conducting and composite bodies [4]-[5], few researches have investigated its use in problems with discontinuous equivalent currents. To address such fact, the electromagnetic radiation from a circular horn antenna is investigated. Simulations are conducted with the CFIE solved by an approach of the MoM slightly different from the Galerkin approach, proposed and discussed in this work.

II. Allocation of TBF and TTF

In this work, the surface equivalent currents **J** and **M** are expanded by triangular basis functions (TBF), $\mathrm{T_j}$, as:

$$\mathbf{J} = \sum_{\mathrm{j}=1}^{N}\left(\mathrm{T_j}(\mathbf{r}')/\rho'\right)I_{\mathrm{j}}^{J} \qquad \text{and} \qquad \mathbf{M} = \sum_{\mathrm{j}=1}^{N}\left(\mathrm{T_j}(\mathbf{r}')/\rho'\right)I_{\mathrm{j}}^{M} \quad (5)$$

where N is the number of TBF and I_{j}^{J} and I_{j}^{M} are unknown coefficients. In the Galerkin approach, triangular test functions (TTF) are employed for the MoM solution. The integrals appearing in the MoM matrix elements are evaluated by Gaussian quadratures with an appropriate singularity extraction [5].

To illustrate the allocation of TBF and TTF, the circular corrugated horn illustrated in Fig. 2 is analyzed. Inside the waveguide horn, at its end-wall at z=0.08λ, electric ($\mathbf{J}_g$) and magnetic ($\mathbf{M}_g$) surface current sources are imposed to excite the fundamental TE_{11} mode. At the same wall, an impedance boundary condition (IBC) is imposed to work as a perfect absorber for the returned TE_{11} mode. Consequently, at the end-wall there are equivalent **J** and **M** currents related to each other by the IBC, as illustrated in Fig. 3(a).

The proposed allocation of TBF is illustrated in Figs. 3(b) through 3(e). The components $\hat{t}$ and $\hat{\phi}$ of $\mathbf{J}_g$, $\mathbf{M}_g$ and **M** (J_g^t, J_g^ϕ, M_g^t, M_g^ϕ, M^t and M^ϕ) are continuous and their TBF are placed over the waveguide end-wall surface, z=0.08λ, as shown in Figs. 3b) and 3c), respectively. The current **J** has a particular behavior at the corner of the end-wall (point ρ=0.31λ and z=0.08λ). The component $\hat{t}$ of **J** (J^t) is continuous while the component $\hat{\phi}$, J^ϕ, is discontinuous. The proposed allocation of their TBF is illustrated in Figs. 3d) and 3e). The continuity of J^t is represented by an entire triangle and the discontinuity of J^ϕ component is imposed by a half triangle in the corner of the end-wall. This distribution of TBF intendeds to ensure the required boundary conditions at this point.

The TTF allocation is illustrated in Fig. 3d). Entire triangles are used at ρ=0.31λ and z=0.08λ. This solution means to impose boundary conditions for CFIE in all segments (points) in the same way (using entire TTF). But this strategy makes the number of base and test function different in the segments where the J^ϕ discontinuity exists. Therefore, the Garlekin technique is not employed and the MoM full matrix is not symmetric.

III. NUMERICAL RESULTS

To demonstrate the effectiveness of the developed strategy, the radiation from the circular horn of Fig. 2 [6] was simulated. The radiation patterns obtained, using the Proposed Approach (PA) and the Galerkin Approach (GA), are illustrated in Fig. 4 together with measured data. When the developed strategy is adopted, the simulated and measured results have an excellent agreement.

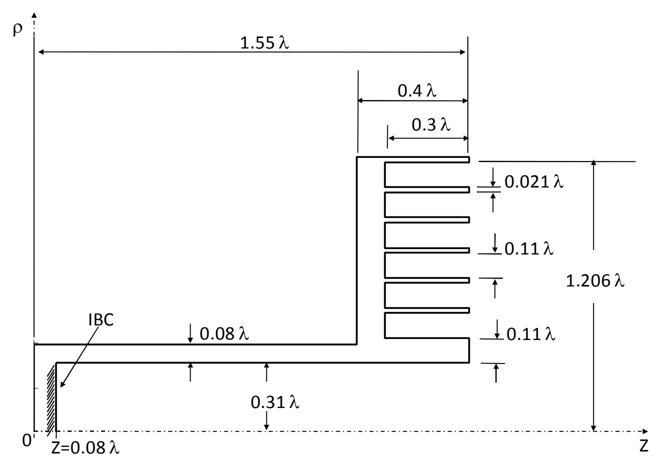

Fig. 2.Circular corrugated horn [6].

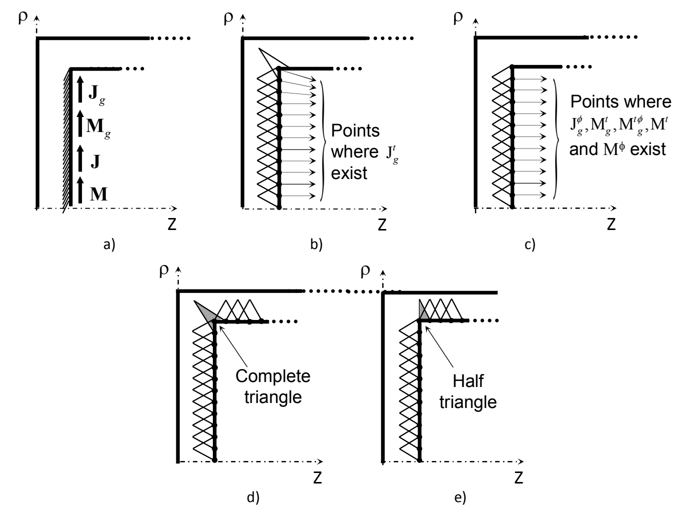

Fig. 3. a) Circular horn waveguide a) Currents allocation. b) J_g^t TBF c) J_g^ϕ, M_g^t, M_g^ϕ, M^t and M^ϕ TBF d) J^t TBF and TTF e) J^ϕ TBF.

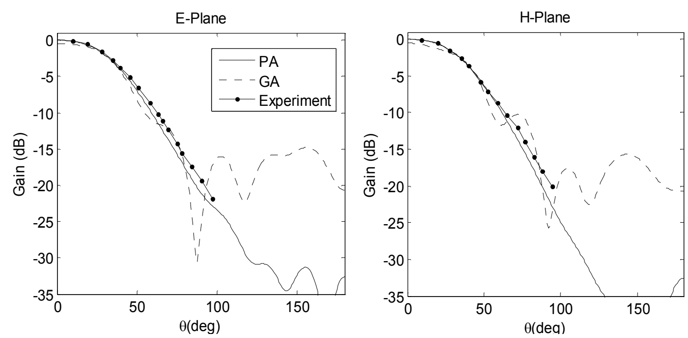

Fig. 4.Circular horn: radiation pattern (PA – Proposed Approach, GA – Galerkin Approach).

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IV. REFERENCES

[1] J. R. Mautz and R. F. Harrington, "H-field, E-field, and combined-field solutions for conducting bodies of revolution," *International Journal of Electronics and Communications*, vol. 32, no. 4, pp. 157-164, 1978.

[2] U. C. Resende, F. J. S. Moreira, and O. M. C. Pereira Filho, "EMFIE and MEFIE formulations for the analysis of scattering from dielectric and composite bodies of revolution," *Microwave and Optical Technology Letters,* vol. 53, no. 2, pp. 398-402, February 2011.

[3] L. Gurel and O. Ergul, "Singularity of the Magnetic-Field Integral Equation and its Extraction," *IEEE Antennas and Wireless Propagation Letters*, vol. 4, issue 1, pp. 229-232, 2005.

[4] P. L. Huddleston, L. N. M. Mitschang and J. M. Putnam, "Combined field integral equation formulation for scattering by dielectrically coated conducting bodies," *IEEE Trans. Antennas Propagat.*, vol. 34, no. 4, pp. 510-520, April 1986.

[5] U. C. Resende and F. J. S. Moreira, "Numerical Convergence of Method of Moments in the Analysis of Bodies of Revolution," 17th International Conference on the Computation of Electromagnetic Fields (COMPUMAG), Florianópolis, SC, Brazil, pp. 817-818, Nov. 2009.

[6] K. A. Iskander, L. Shafai, A. Frandsen, and J. E. Hansen, "Application of impedance boundary conditions to numerical solution of corrugated circular horns," *IEEE Trans Antennas Propagat* AP-30, 366–372, 1982.